A Gaussian Derivative Model of the Complex Cell

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1. Introduction

- Visual filters can be modelled by derivatives G_k of the Gaussian function.
- This Gaussian jet representation is convenient because it is:
  - Steerable, hence all orientations can be represented concisely.
  - Dimensionally separable, hence easily defined in 2D and 3D.
  - The natural code for typical image features, e.g. edges and blobs.
- But what about complex cells, cf. the Gabor energy model?
- Can the jet be made insensitive to small shifts of the image?

2. Subunit Filters

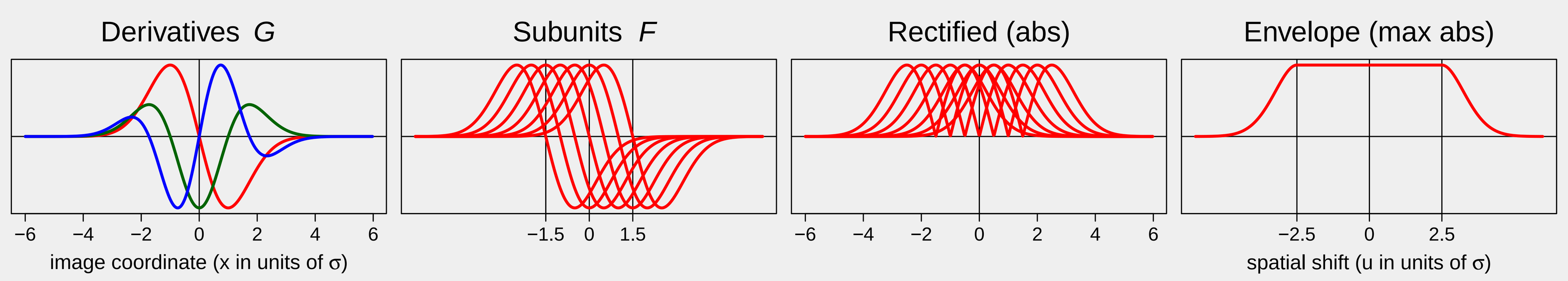
- Let $F_\star(x, u)$ be a family of subunit filters, parameterized by shift u.
- These can be Taylor-approximated from $F_\star(x, 0)$ and its derivatives.
- In particular, choose the edge-filters $F_\star(x, u) = G_1(x - u, \sigma)$.
- Approximate filters $F \approx F_\star$ can be obtained from the Dth-order jet:

$$F(x, u) = \sum_{k=0}^{D-1} \frac{(-u)^k}{k!} G_{k+1}(x, \sigma)$$

- Problems: Unstable, and nature of the approximation is unclear.
- Solution: Allow polynomial weights $P_k(u)$, and solve by least-squares.

3. Invariant Response

- The basis G is used to synthesize the filters F_i, for each shift u_i.
- Each subunit filter is applied to the signal s, giving $q_i = F_i \cdot s$.
- The response-envelope is computed by the operation $\max_i |q_i|$.



- More derivatives are needed in practice (see box 6).
- The range of shifts must cover $\pm\sigma$ for a unimodal impulse-response.
- Note that the family of subunit filters is continuous (only 7 shown).

4. Neural Implementation

- A neurally plausible 'softmax' is used to compute the response-envelope:

$$\max_i |q_i| \approx \textstyle\sum_i w_i |q_i|$$

- The weights w_i are defined by a nonlinearity and normalization:

$$w_i = \exp\big(\mu |q_i|\big) \Big/ \textstyle\sum_i \exp\big(\mu |q_i|\big)$$

5. Matrix Formulation

- F : Subunit filters (rows) $\quad$ P : Polynomials (columns)
  G : Gaussian derivatives (rows) $\quad$ M : Monomials (columns)
  s : Input signal (column) $\quad$ C : Estimated coefficients
- The subunit-filters are polynomially-weighted Gaussian derivatives:

$$F = PG \quad \text{where} \quad P = MC$$

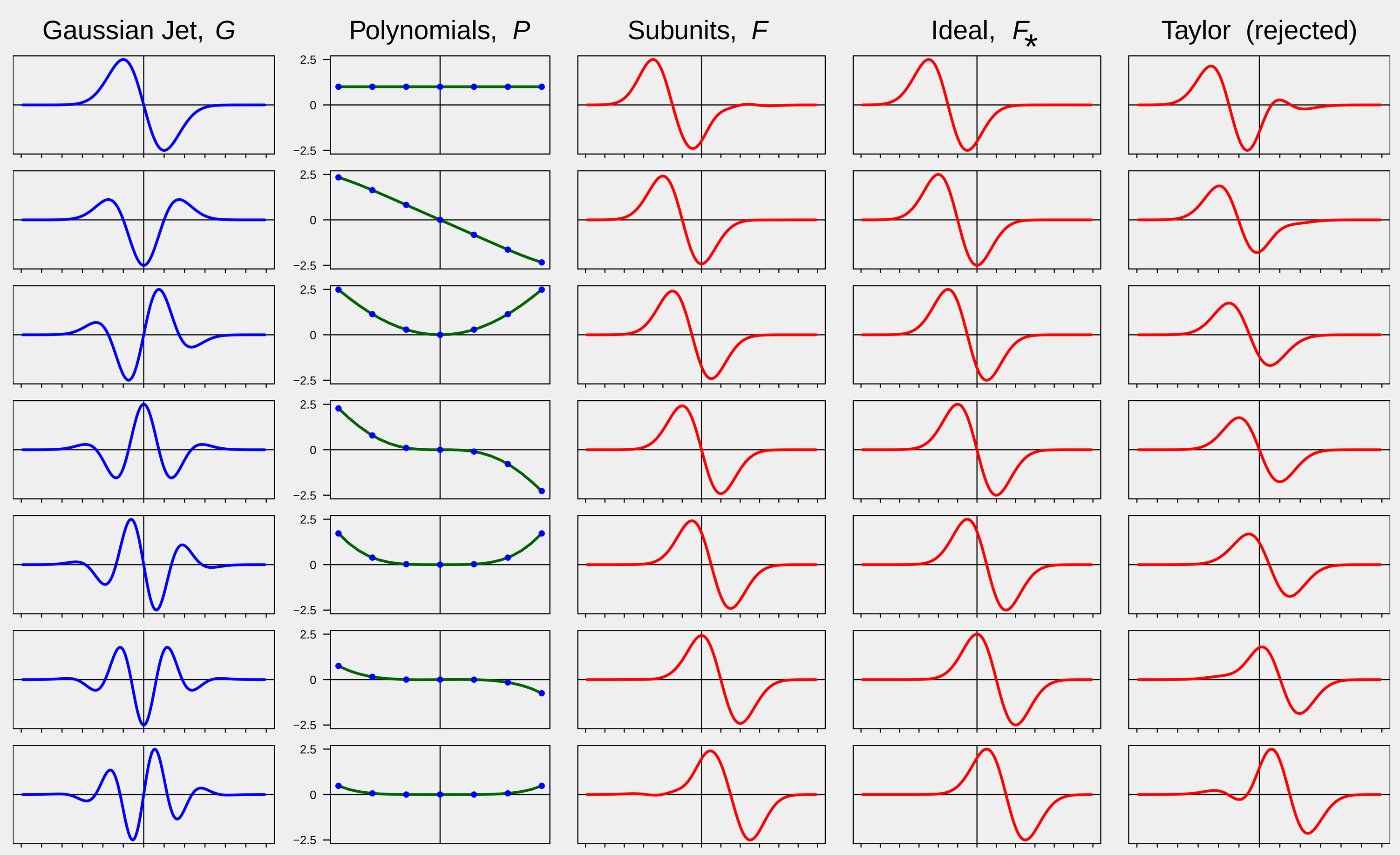
- The filter-design problem is to estimate C, given ideal filters $F_\star$.
- The least-squares solution involves two pseudo-inverses:

$$F = MCG \quad \text{where} \quad C = M^+ F_\star G^+$$

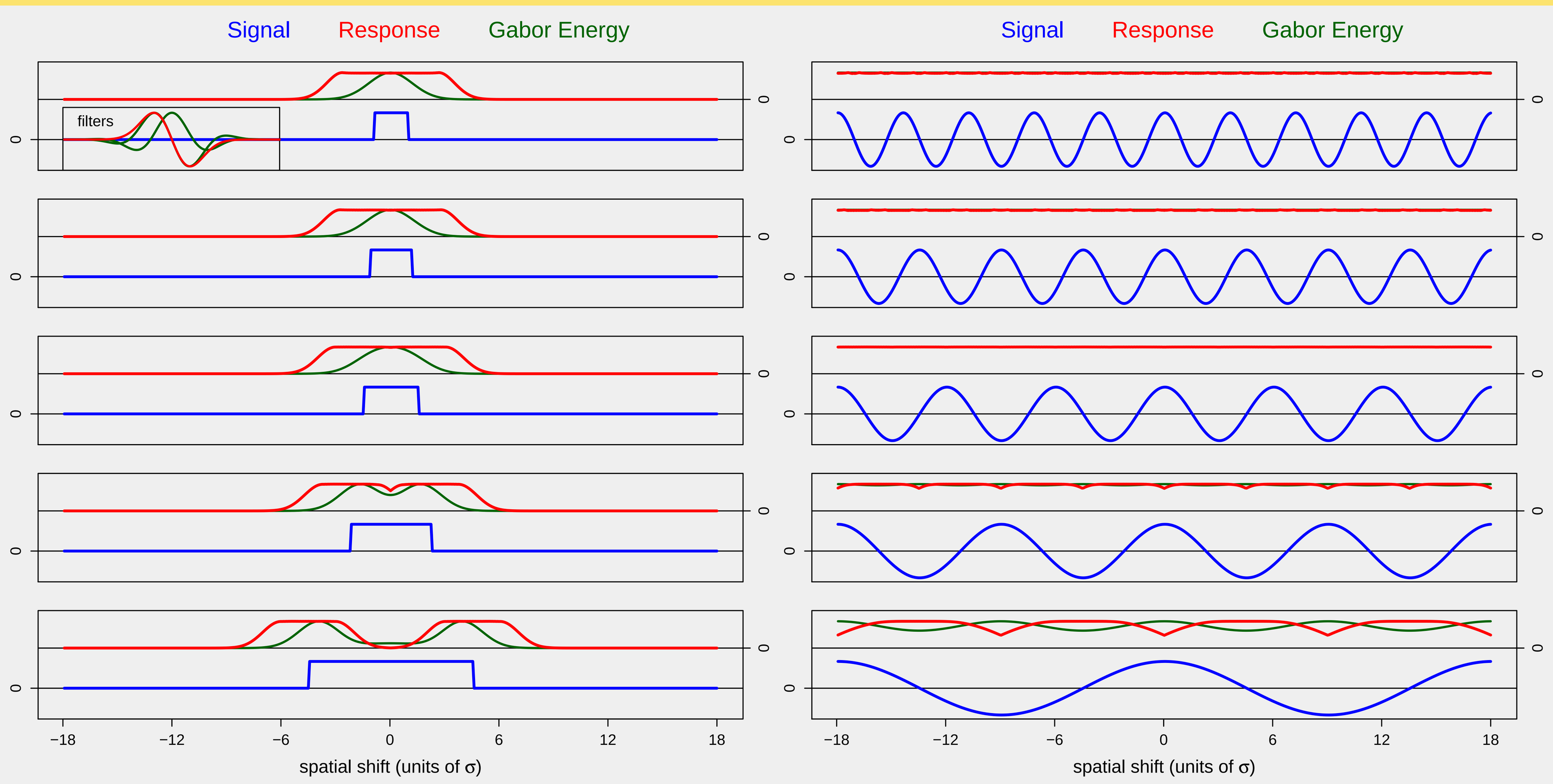
- The subunit response q is a linear transformation of the jet response:

$$q = Fs = P(Gs)$$

6. One-Dimensional Example



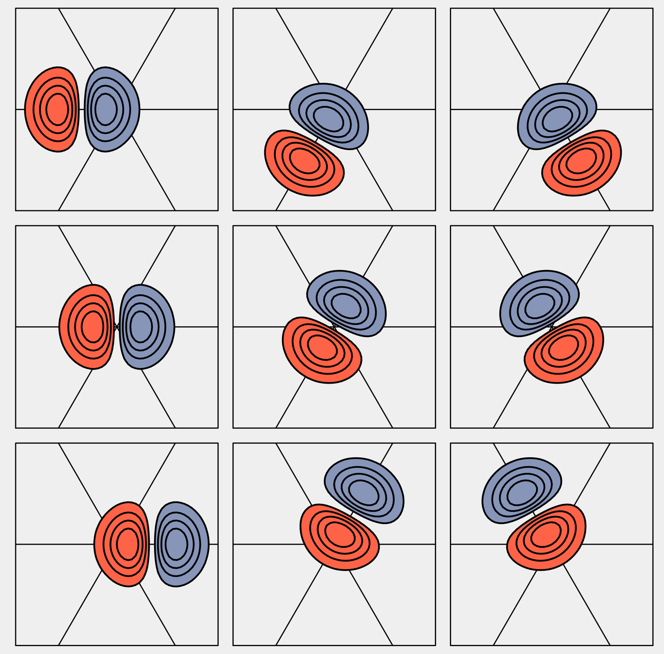
7. Bar and Grating Responses



8. Steerability

- Any Gaussian derivative $G_k(x, \sigma, \theta)$ can be exactly synthesized from the basis $G_k(x, \sigma, \theta_j)$, where $j = 1 \cdots k + 1$.
- A complete basis of order D requires $\sum_{k=1}^{D}(k+1) = \frac{1}{2}D(D+3)$ filters.
- The basis matrix G must be extended, but there is otherwise no change to the filter-synthesis algorithm (box 5).

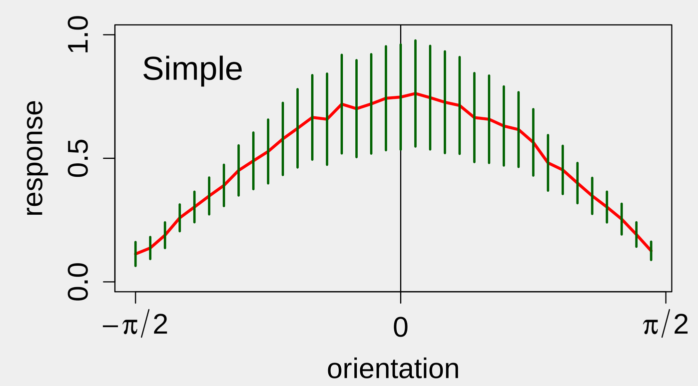
9. Two-Dimensional Example

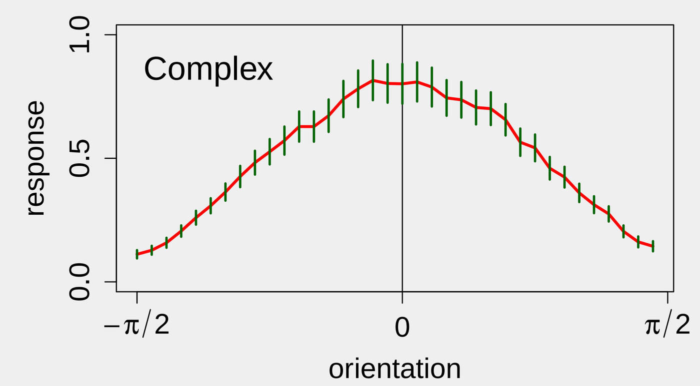


- The complete basis of order $D = 8$ contains 44 oriented filters, all centred at the same location.
- This permits accurate synthesis of any filter $G_1(x - u, \sigma, \theta)$, where $u \in \pm 1.5\sigma$ and $\theta \in [0, 2\pi)$.
- Each column corresponds to a complex cell RF, $\theta = 0°, 60°, 120°$.
- Each box shows a subunit.

10. Natural Image Response

- The model is evaluated in the framework of 'slow feature analysis'.
- Pick 100 points at random, from an image with a dominant orientation.
- Choose straight 'tracks' at 36 orientations through each point.
- Apply simple/complex cell model at 100 points along each track.
- Compute mean response and SD in each orientation, over all points.





- The simple response is orientation tuned (red curve), but highly variable.
- The complex response is also orientation tuned, but much less variable.

11. Conclusions

- A shift-insensitive response can be obtained from the Gaussian jet.
- The signal structure can be represented geometrically.
- The new model is steerable, and works in any number of dimensions.
- High-order filters, as seen in neural data, are needed in the jet basis.

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